Poisson Surface Reconstruction

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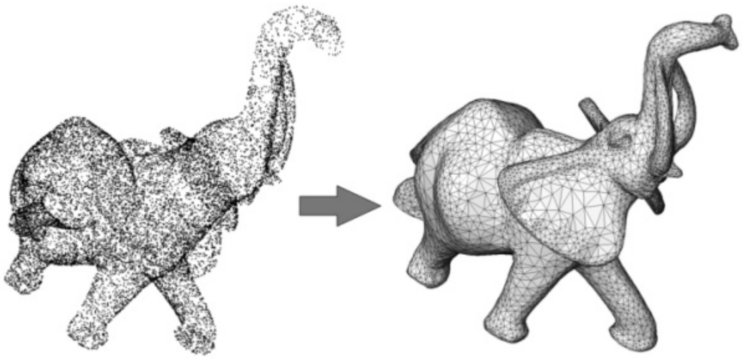
Surface Reconstruction

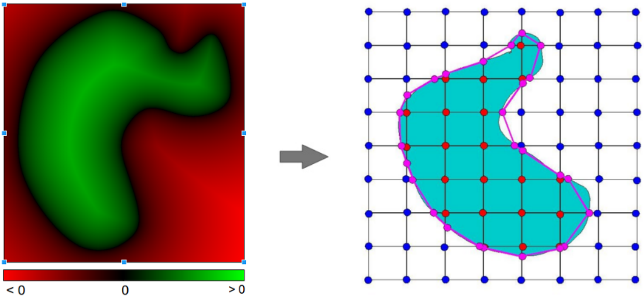
Generate a mesh from a set of surface samples



Left: 17K points sampled on the statue of an elephant with a Minolta laser scanner. Right: reconstructed surface mesh.

Implicit Function Approach

1. Find the indicator function which has values less than zero outside the model and greater than zero inside
2. Use a contouring algorithm, e.g. marching cubes, to find zero set



Poisson Reconstruction

Input:

- A point cloud
- Oriented normals at each point

Output:

- The indicator function

Pros:

- Robust as it solves a well-posed sparse Poisson problem
- Resilient to noise as it processes all the points globally

Cons:

- Requiring oriented normals
- Capturing certain holes requires additional human input

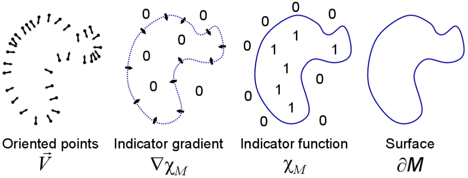
Main Idea

The gradient of the indicator function is equal to the field defined by the surface normals near the surface, and zero elsewhere:

$$\nabla \chi = \vec{V}$$

Cast into a minimization problem:

$$\nabla^2 \chi = \nabla \cdot \vec{V}$$

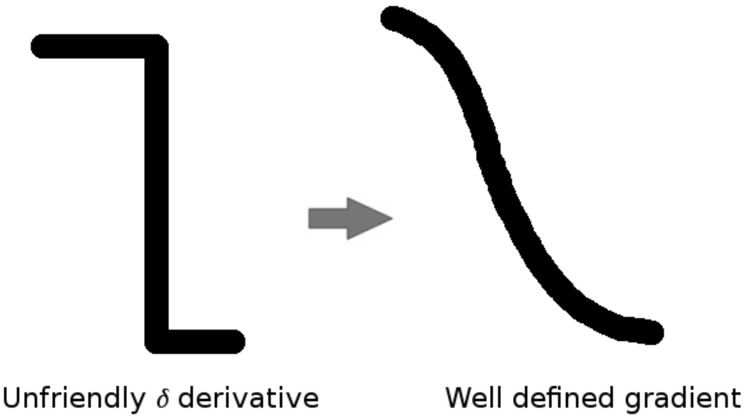


Numerical Method

Assumption: Uniform sample density

- Definition of smoothed χ
- Three-dimensional Grid
- Numerical Solution Function Space
- Finding smoothed $\vec{V}$
- Discretizing $\nabla^2 \chi = \nabla \cdot V$ into $Lx = v$
- Solving the linear system of equations

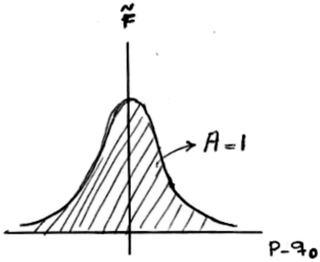
Smoothing the indicator function



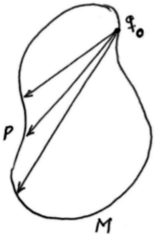
Smoothing the indicator function

$$(\chi * \tilde{F})(q_0) = \int_M \tilde{F}(p - q_0)\chi(p)dp$$

$$\nabla(\chi * \tilde{F})(q_0) = \int_M \tilde{F}(p - q_0)\vec{N}(p)dp$$



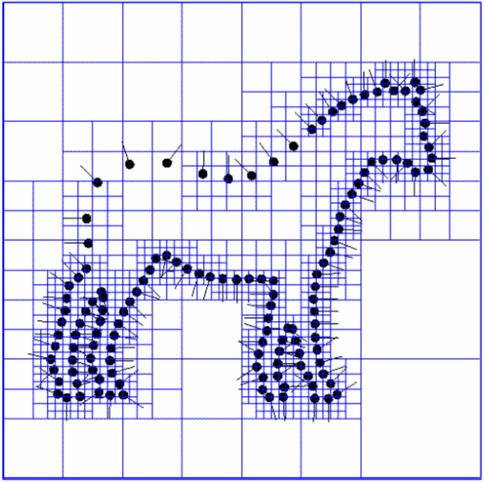
Schematic of the filter function



Integration over all the damin

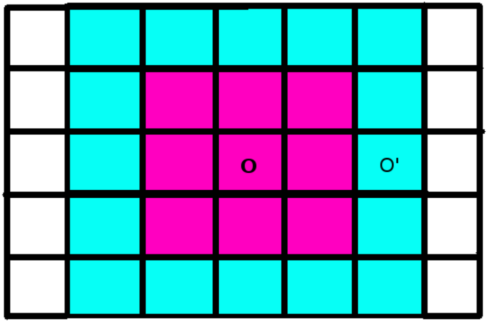
Three-dimensional Grid

- Octree $\mathcal{O}$ with prescribed depth D.
- Each sampling point must lie inside a depth D cell.



Basis Functions

- For every cell $o \in \mathcal{O}$, a basis function is defined as $F_o(q) = \frac{1}{o.w^3}\phi(\frac{q - o.c}{o.w})$
- ϕ is a quadratic function approximating a Gaussian with unit variance.
- $F_o(p)$ will be used as the filter function too, i.e., $\tilde{F}(p - o.c) = F_o(p)$.



Solution Space

The numerical solution will be defined as:

$$X(p) = \sum_{o \in \mathcal{O}} \chi_o F_o(p)$$

The final goal is to find the discrete values χ_o for every octree cell.

Finding $\vec{V}$

Original Definition:

$$\vec{V}(q_0) = \int_M \tilde{F}(p - q_0)\vec{N}(p)dp$$

Numerical Approximation ($\mathcal{S}$ is the set of sample points):

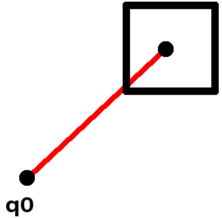
$$\vec{V}(q_0) = \sum_{s \in \mathcal{S}} \tilde{F}(s - q_0)s.\vec{N}\mathcal{P}_s$$

Ignoring the constant surface area and using trilinear interpolation:

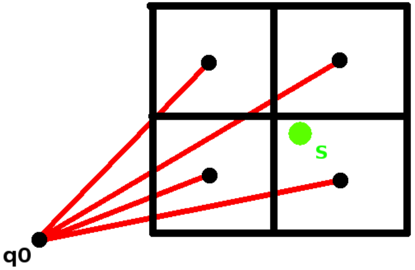
$$\vec{V}(q_0) = \sum_{s \in \mathcal{S}} \sum_{o \in \mathcal{N}_D(s)} \alpha_{o,s} F_o(q_0)s.\vec{N}$$

Finding $\vec{V}$

$$\tilde{F}(s - q_0) = \sum_{o \in \mathcal{N}_D(s)} \alpha_{o,s} F_o(q_0)$$



Simple case when s=o



Trilinear interpolation

Solving for X_o values

Similar to the Finite Element Methods, we seek the values of X_o to minimize:

$$\sum_{o \in \mathcal{O}} |\langle \nabla^2 X - \nabla \cdot \vec{V}, F_o \rangle|^2$$

Which reduces to minimizing the following norm:

$$\|Lx - v\|_2$$

$$v_o = \int_M F_o(q) \nabla \cdot \vec{V}(q) dq$$

$$L_{oo'} = \int_M \nabla \cdot F_o(q) F_{o'}(q) dq$$

Multigrid Method

Ideal procedure when dealing with:

- Huge system of linear equations
- Arising from the discretization of a Poisson problem
- On a hierarchical structured grid
- Rank deficient left hand side (no boundary conditions).

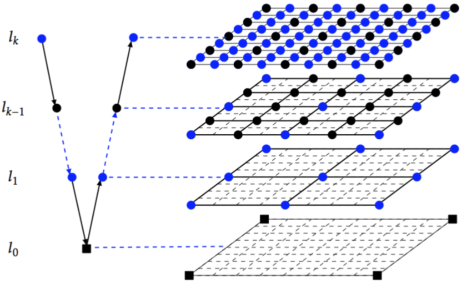
Catch:

- Difficult to implement

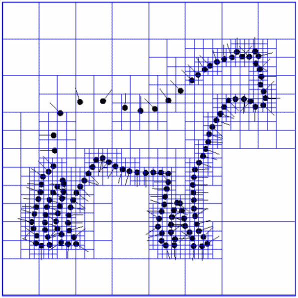
Simple Jacobi iteration as smoother:

$$x^{(k+1)} = v - Lx^{(k)}$$

Restriction, prolongation, and multigrid cycles:

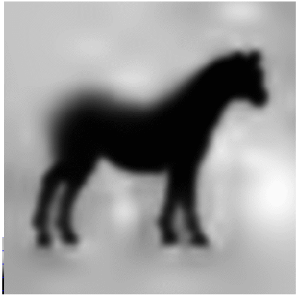


Schematic Example



Sample points and octree

Source Term (∇ V)

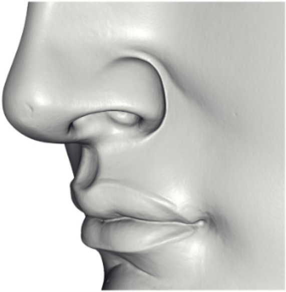


Indicator Function (X)

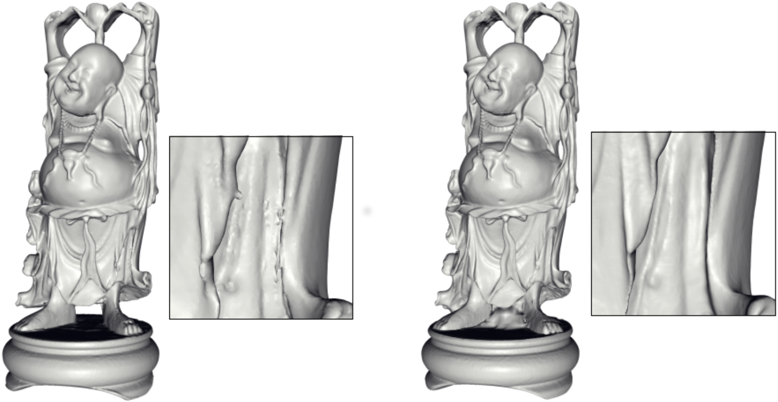
Michael Angelo's David

- 215 million data points from 1000 scans
- 22 million triangle reconstruction
- Compute Time: 2.1 hours
- Peak Memory: 6600MB

Michael Angelo's David



Comparison to VRIP



VRIP | Poisson

Conclusion

- Advantages:
  - Resilient to noise
  - Considers all sample points simultaneously
  - Can be done in parallel or out-of-core
- Issues:
  - Requires further post-processing for capturing certain features
  - Requires sample point normals as a priori
  - Resulting surface does not necessarily contain any of the sample points
  - Not much is mentioned about creases