Physics Integration via Regression

Shayan Hoshyari, Chenxi Liu

April, 2018

Motivation

Physics simulations can sometimes be cast as regression problems.

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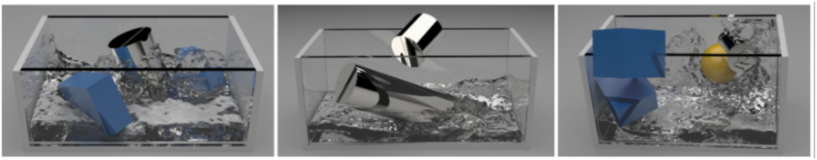
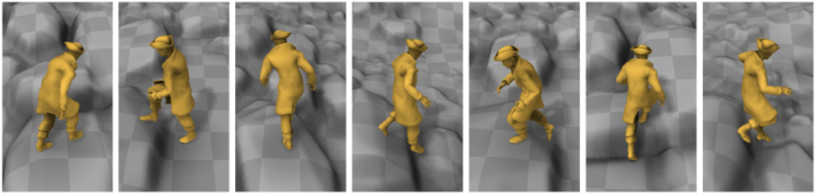


[Ladicky, et. al. SIGGRAPH Asia 2015]

Physics simulations can sometimes be cast as regression problems.

- Speed.
- Only data is available.



[Holden, et. al. SIGGRAPH 2017]

The problem

- Dynamical system, $\dot{\mathbf{s}} = \mathbf{f}(t, \mathbf{s}, \mathbf{u})$

  $\mathbf{s}$: state variables

  $\mathbf{u}$: control inputs

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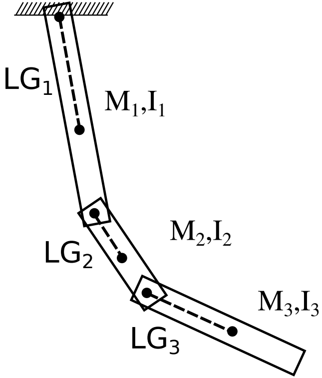
- Example, three link pendulum:



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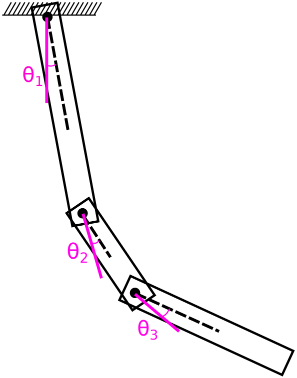

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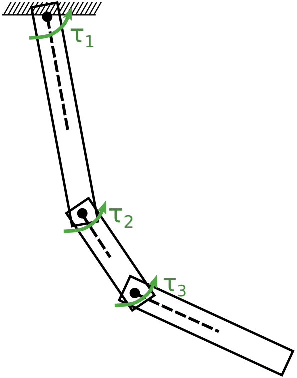

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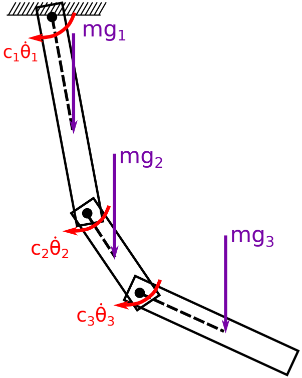

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Other forces included in $\mathbf{f}$

The problem – continued

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Grzeszczuk, Radek, Demetri Terzopoulos, and Geoffrey Hinton. SIGGRAPH 1998.

Given a pendulum with fixed properties (fixed $\mathbf{f}$),

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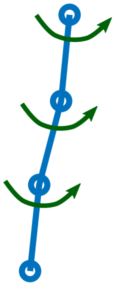
- Implement a control algorithm,

  i.e., solve for $\mathbf{u}$ in the inverse problem:

  $\dot{\mathbf{s}} = \tilde{\mathbf{f}}(t, \mathbf{s}, \mathbf{u}), s = s(t)$

Learning problem

- Input: $\mathbf{y} = [\mathbf{s}_0, \mathbf{u}_0]$



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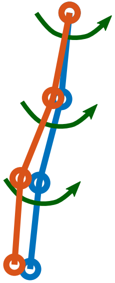

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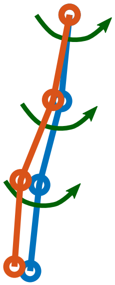

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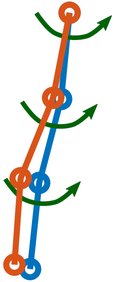

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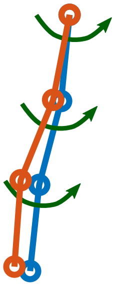
- Sampling:

```
Y = [rand(n, 3)*r1 rand(n, 3)*r2 rand(n, 3)*r3]
```



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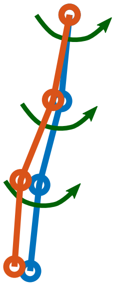
- Paramters: Δt, r_1, r_2, r_3.



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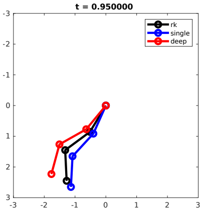
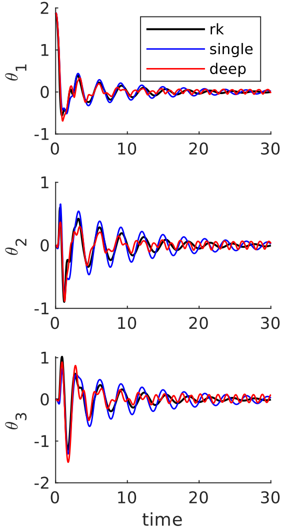
  `minimize.m` by Carl Edward Rasmussen.

Results – no **u**

```
n_links=3; M=[1,1,1]; L=[1,1,1]; LG=L; g=10; I=[0,0,0]; c=[1,1,1];
n=5000; r1=2pi; r2=2;
```

Arch.	Train error	Test error
deep	4.13%	4.75%
single layer	7.35%	7.72%





Results – with **u**

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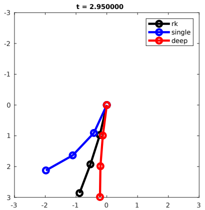
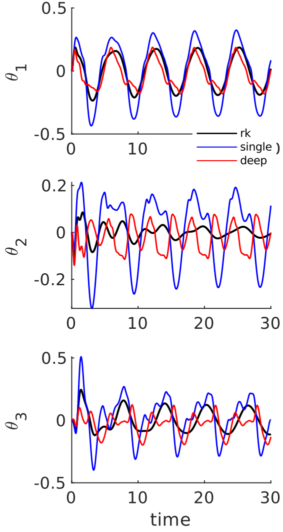
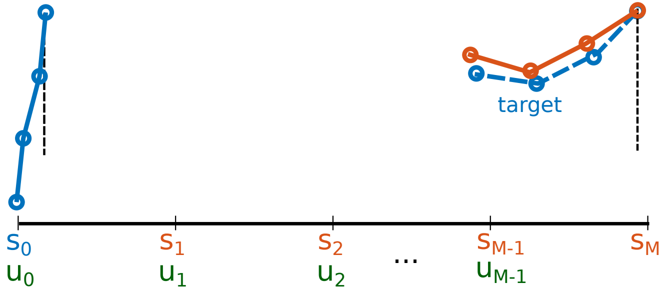
n_links=3; M=[1,1,1]; L=[1,1,1]; LG=L; g=10; I=[0,0,0]; c=[1,1,1];
n=5000; r1=2pi; r2=2; u=[5*cos(t),0,sin(t)]
```

Arch.	Train error	Test error
deep	7.23%	7.97%
single layer	10.57%	11.97%





Control problem



- s_0, target
- $\mathbf{U} = [\mathbf{u}_0, \mathbf{u}_1, \ldots, \mathbf{u}_{M-1}]$
- $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, \ldots, \mathbf{s}_M]$
- Control Problem:

  $\arg\min_{\mathbf{U}} E(\mathbf{U}; \text{target}, s_0) = \mu \|\mathbf{U}\|_2^2 + (s_M - \text{target})^2$

Gradient of control objective, $\nabla_U E$

Similar backpropagation problem as deep neural networks:

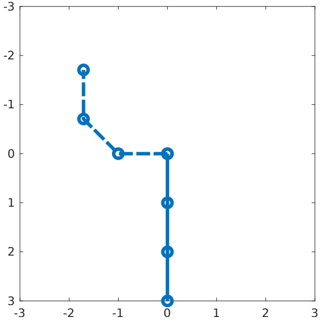
$$\begin{bmatrix} \boldsymbol{I} & \boldsymbol{I} + \partial_{\mathbf{s}_1} \mathbf{C}_2 & & & \\ & \ddots & & & \\ & & \boldsymbol{I} & \boldsymbol{I} + \partial_{\mathbf{s}_j} \mathbf{C}_{j+1} & \\ & & & \ddots & \\ & & & & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \partial_{\mathbf{s}_1} \mathbf{s}_M \\ \vdots \\ \partial_{\mathbf{s}_j} \mathbf{s}_M \\ \vdots \\ \partial_{\mathbf{s}_M} \mathbf{s}_M \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ \boldsymbol{I} \end{bmatrix}$$

$$\partial_{\mathbf{u}_j} \mathbf{s}_M = \partial_{\mathbf{s}_{j+1}} \mathbf{s}_M \left(\partial_{\mathbf{u}_j} \mathbf{C}_{j+1} + \boldsymbol{I} \right)$$

Control Results

Easier pendulum:

```
n_links = 3;
M = [1 1 1];
L = [1 1 1];
LG = [1, 1, 1];
g = 10;
I = [0,0,0];
c = [1, 1, 1];
```



More complicated physics:

```
n_links = 3;
M = [1 1.5 2];
L = [1 1.5 2];
LG = L ./ 2.;
g = 10;
I = 1/12 .* M .* L .* L;
c = [1, 1, 1];
```